CALCULATION OF FLIGHT DIRECTIONS OF BIRDS OBSERVED CROSSING THE FACE OF THE MOON

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THE classic paper of Lowery (1951) on the study of nocturnal migration by observing birds passing in front of the moon has established this technique as an important part of the field study of migration. Most migration students are unable to use it, however, because of the mathematical calculations necessary to reduce the observational data. The purpose of this paper is to provide instructions by which any observer, armed only with a telescope, a compass and a protractor, may perform these calculations without technical assistance.

The observational method and the theory of the calculation are described by Lowery at considerable length, and reference should be made to his paper for more details than can be given here (see also Appendix). The chief difficulties arise from the oblique angle at which the birds are viewed, which distorts the direction in which they are actually flying. This effect becomes more serious as the moon becomes low in the sky, and when it is close to the horizon the birds' flight directions cannot be determined at all. Even if observations are avoided in these circumstances, a mathematical treatment is usually necessary before directions of flight can be determined with any accuracy.

Lowery avoided this difficulty in part by processing the data from all his collaborators himself, but this procedure was so time-consuming that he was forced to adopt a method of approximation, grouping the data before making the calculation. However, Tunmore (1956) has recently pointed out that Lowery's approximation might lead to large errors if it should be used when the moon is low in the sky. A method of grouping will be described here which introduces errors no larger than those inherent in the observational method itself.

OBSERVATIONS

Position of the Moon.—To reduce the data it is necessary to know the compass direction and altitude (angle of elevation) of the moon at the time that each bird is seen. As Lowery explains, this information can be obtained from published tables, but these tables are not generally available and it is much simpler to measure the angles directly.

The compass direction should be measured in degrees (North, 0° or 360°; East, 90°; South, 180°; West, 270°; etc.). A correction is added or subtracted for the difference between true north and magnetic north (in the United States, for example, the local correction is given on the topographic maps published by the U. S. Geodetic Survey), and the result is the azimuth angle Z.

The altitude is measured by attaching a protractor, with a plumbline (thread and small weight) hanging *freely* from its *exact* center, to the side of the telescope so that the base of the protractor is exactly parallel to the line of sight (see Fig. 1). When the moon is in the center of the telescope field the position of the thread on the protractor scale is read and the reading subtracted from 90°; this gives the altitude A in degrees.

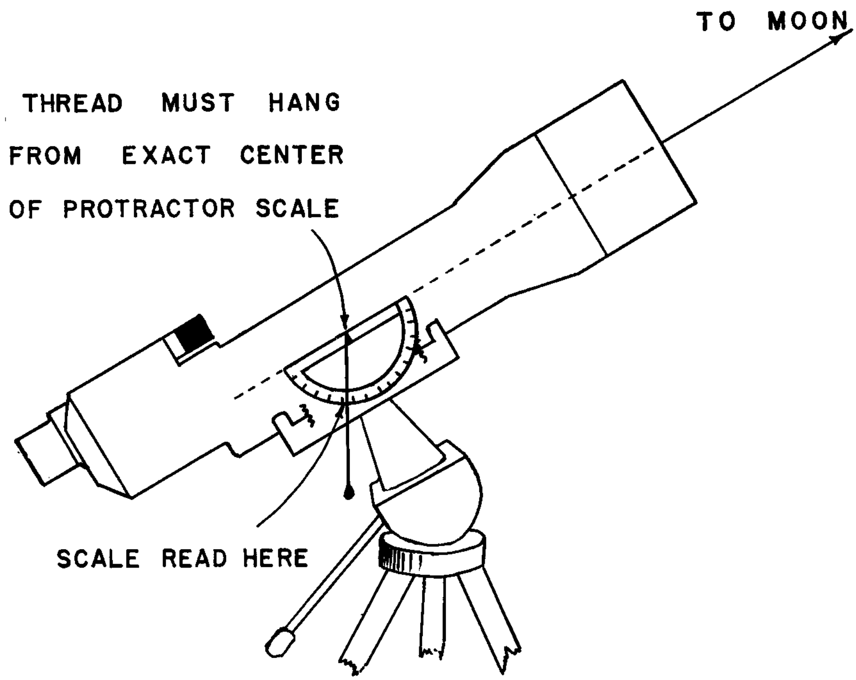

FIG. 1. Attachment of protractor to telescope.

Each of these measurements should be made at frequent intervals, about once every half hour and at least four times in all during the observation period. At the end of the period each of the angles is plotted against the time of the reading on a piece of graph or squared paper, and smooth curves are drawn through the points, averaging out small errors in measurement. It should then be possible to read off from the curves the exact azimuth and altitude of the moon at any intermediate time, within five degrees for the former, and two degrees for the latter.

If the altitude of the moon is less than 14° the calculation should not be made at all: observations made in these circumstances have only a qualitative value.

Apparent Directions of Flight.—Observations are best made with a low power telescope (15× to 20×) on a rigid tripod mounting; the latter is essential, not only for comfort, but also for the measurement of the altitude which was described above. At least two collaborators are required, one to observe and one to record the data, and positions should be changed frequently to avoid fatigue and eyestrain. The period for which the moon is kept under continuous scrutiny must be recorded exactly, with note of any interruptions, such as those which occur when the observers change positions.

The apparent flight direction of each bird is estimated by visualizing a clock-face on the moon, with 12 o'clock at the top. The "hours" at which the bird appears to enter and leave the moon are noted, to the nearest half-hour unless the bird is flying more or less "vertically" (e.g., from 5 o'clock to 2), when it is sufficient to record to the nearest hour. This information is recorded for each bird, together with the time to the nearest minute and any other relevant information.

(The observational method recommended here is exactly that described by Lowery, and his paper should be consulted for fuller details.)

Calculations

The method of calculating the flight direction of a single bird will be outlined first, and the grouping of the data discussed later.

Flight Direction of a Single Bird.—(1) The "hours" at which the bird entered and left the moon are used to obtain an angle B from Table 1. This is the angle which the bird appears to make with the horizontal, and the remainder of the calculation consists in its correction for the effect of foreshortening.

(2) If B is not 0° or 90°, the moon's altitude A at the time of the observation is determined from the graph, to the nearest 2° if less than 22°, to the nearest 5° if between 25° and 50°, to the nearest 10° if over 50°. From the two angles B and A a new angle C is obtained from Table 2. If B is 0° or 90° there is no correction and C is the same as B.

(3) The moon's azimuth Z at the time of the observation is obtained from the second graph.

(4) It is now necessary to note whether the bird appeared to be flying upwards or downwards, and whether to the left or right (e.g., 8 o'clock to 2 is up and right, 2 to 8 is down and left). Four possibilities arise:

Up and right: add 90° to Z and add C.

Down and right: add 90° to Z and subtract C.

Down and left: subtract 90° from Z and add C.

Up and left: subtract 90° from Z and subtract C.

The result is the true flight direction of the bird in degrees (if negative, 360° is added; if over 360°, 360° is subtracted).

TABLE 1

DETERMINATION OF THE ANGLE B, THE APPARENT DIRECTION OF FLIGHT

Hour of entering:	12	1	2	3	4	5	6	7	8	9	10	11
Hour of leaving												
12	X	15	30	45	60	75	90	75	60	45	30	15
1	15	X	45	60	75	90	75	60	45	30	15	0
2	30	45	X	75	90	75	60	45	30	15	0	15
3	45	60	75	X	75	60	45	30	15	0	15	30
4	60	75	90	75	X	45	30	15	0	15	30	45
5	75	90	75	60	45	X	15	0	15	30	45	60
6	90	75	60	45	30	15	X	15	30	45	60	75
7	75	60	45	30	15	0	15	X	45	60	75	90
8	60	45	30	15	0	15	30	45	X	75	90	75
9	45	30	15	0	15	30	45	60	75	X	75	60
10	30	15	0	15	30	45	60	75	90	75	X	45
11	15	0	15	30	45	60	75	90	75	60	45	X

Note: directions corresponding to "half-hours" can be obtained by interpolation.

Accuracy.—The accuracy of the calculated flight direction may be estimated from Table 2. Assuming that A and B are correctly observed to the nearest figure given in Table 2, and that Z is measured correctly to the nearest multiple of 5°, the following estimates are obtained for the maximum possible error introduced by the approximations used in the calculation:

Moon's altitude A:	14°	16°	20°	30°	40°	60°
Max. error in direction:	20°	18°	15°	13°	12°	12°

The probable error of each calculated direction will be roughly one-third of the maximum error given in the above table.

Two other sources of error are not included in the above estimates: the possibility that the bird may not be flying exactly horizontally (which would lead to error in determining B), and errors in observation. Both may become serious as the moon becomes low in the sky—a mistake of "half an hour" in recording the apparent direction of flight, for example, leads to an error in the calculated flight direction which approaches 30° if the altitude of the moon falls below 15°—and the results should always be evaluated with this in mind.

TABLE 2
DETERMINATION OF THE ANGLE C

Moon's altitude A	Apparent direction of flight B						
	7½	15	22½	30	45	60	75
14	29	48	60	67	76	82	86
16	26	44	56	64	75	81	86
18	23	41	53	62	73	80	85
20	21	38	50	59	71	79	85
22	19	36	48	57	69	78	84
25	17	32	44	54	67	76	84
30	15	28	40	49	63	74	82
35	13	25	36	45	60	72	81
40	12	23	33	42	57	70	80
45	11	21	30	39	55	68	79
50	10	19	28	37	53	66	78
60	9	17	26	34	49	63	77
70	8	16	24	32	47	62	76

Note: if the moon's altitude is greater than 70° the birds are almost vertically overhead, and their flight directions should be estimated directly.

Grouping of the Data.—Once the above calculation has been carried out a few times, it should be possible to perform it quickly and accurately. It is then possible to reduce the labor of analysis by judicious grouping of the data. I have found useful the following method of grouping. The data sheet is ruled into columns, with one column devoted to each step in the calculation, and one line to each bird. Two columns record, for easy reference, the variation of the azimuth Z and altitude A of the moon through the observation period. Then a single calculation suffices for all the birds with the same apparent angle of flight B and the same values of A (within the accuracy demanded by Table 2) and Z (within the nearest five degrees). These criteria usually permit the grouping in 15- or 20-minute periods of all birds flying in the same apparent direction, and the limits of error quoted in the preceding paragraph are not exceeded.

FLIGHT DENSITY

Before the results can be used for quantitative studies of migration, it is necessary to calculate the flight density (e.g., the number of birds crossing a mile front in an hour) from the number of birds seen. However, this calculation requires an assumption to be made concerning the variation of the density with height. Using the scanty data then available, Lowery (1951) assumed for the purposes of calculation that the flight density was uniform with height up to a ceiling of one mile. Recent work by Harper (1958) suggests that this assumption is reasonably good, in that nocturnal migrants in southern England usually fly at heights between 2000 and 5000 feet, and to give figures comparable with Lowery's the same assumption will be used here. However, variations in the average height of migration (such as those found by Harper) would have a serious effect on the calculated flight densities (see Appendix), and until a method is devised to detect such variations the results cannot be quantitatively accurate. Hence, there is no need to perform the calculation with great accuracy.

For each hour of observation, the numbers of birds seen flying in each direction are grouped under 22½° headings, and a correction is made for any gaps in observation to give an estimate of the total number of birds crossing the moon in each 22½° sector per hour. The altitude of the moon in the middle of the hour is noted, and used to obtain a correcting factor from Table 3: if the birds are flying more or less "horizontally" (e.g., 8 o'clock to 4), factor X is used; if they are flying more or less "vertically" (e.g., 1 o'clock to 5), factor Y is used. Approximate correcting factors for oblique directions can be obtained by interpolation. The number of birds seen per hour is multiplied by the correction factor: this gives an estimate of the number of birds crossing a mile front per hour, in the direction concerned (this quantity is termed the "Sector Density" by Lowery).

TABLE 3

CORRECTION FACTORS FOR DETERMINING FLIGHT DENSITIES

Moon's altitude A:	15°	20°	25°	30°	35°	40°	45°	50°	60°	70°	90°
Correction factor X:	15	30	40	60	80	100	120	140	180	210	240
Correction factor Y:	60	80	100	120	140	150	170	180	210	220	240

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SUMMARY

Instructions and tables are given from which observers without mathematical training can calculate flight directions and approximate densities of migrating birds observed flying across the face of the moon. Estimates are given of the accuracy of the results.

The mathematical theory of the calculations is outlined in an Appendix. It is pointed out that the results obtained for the flight density depend critically on the distribution of migrating birds with height, which is not yet known.

APPENDIX: MATHEMATICAL BACKGROUND

The method described above for computing flight directions is derived from an equation equivalent to Lowery's equation (1), namely,

$$\tan(\tfrac{1}{2}\pi + \eta + Z) = \operatorname{cosec} A \tan B,$$

where η is the bird's flight direction, A and Z are the altitude and azimuth of the moon, and B is the apparent direction of the bird's flight across the imaginary clock-face, measured counterclockwise from the line from 9 o'clock to 3. The only assumptions used in deriving this equation are that all the birds are flying horizontally and that the earth is flat.

The density of migration is calculated by assuming a distribution with height h (measured in miles) of $f(h)$ birds per mile front per hour. Thence, assuming that the birds are flying randomly, not in flocks, the correction factors are:

$$X = \frac{120 \sin^2 A \int_0^\infty f(h)\, dh}{\int_0^\infty h f(h)\, dh}; \qquad Y = X \operatorname{cosec} A,$$

under the approximation that the moon subtends an angle of 1/120 radians at the surface of the earth. Using Lowery's simplifying assumption that $f(h)$ is constant up to $h = 1$ mile and zero above this, we obtain:

$$X = 240 \sin^2 A; \qquad Y = 240 \sin A,$$

the values given in Table 3. The correction factors do not depend strongly on the *shape* of the assumed distribution: if, for example, we assume $f(h) = h \exp(-2h^2)$, a more realistic distribution which falls to zero at the ground, peaks at $h = \frac{1}{2}$ mile and drops off rapidly above $h = 1$ mile, they are altered by only 25 per cent. However, if two possible distributions have the same shape but differ in scale-height, the number of birds seen will vary directly with the scale. For example, if the total flight density remains unaltered but the ceiling falls from $h = 1$ mile to $h = \frac{1}{2}$ mile, the number of birds actually seen will be halved, and the estimate of flight density will be wrong by a factor of two. As Lowery points out (1951:389–390), hour-to-hour or night-to-night variations in the average height of the migrating birds cause serious errors in the quantitative estimates of flight density.

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