J. Field Ornithol., 60(4):520–522

MICROCOMPUTER CALCULATION OF DISTANCE AND INITIAL DIRECTION ALONG GREAT-CIRCLE ROUTES

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Abstract.—The shortest distance between two points on the earth's surface is that along a great-circle route. A procedure well-suited to microcomputer programming in BASIC is described for calculating this distance and the initial direction, given the latitude and longitude of each of the two points.

PROGRAMA EN MICROCOMPUTADORA PARA DETERMINAR LA DISTANCIA Y DIRECCIÓN INICIAL A LO LARGO DE "RUTAS DEL CÍRCULO MAYOR."

Resumen.—La distancia más corta entre dos puntos de la superficie de la Tierra (una geodésica) es la distancia medida a lo largo del Círculo Mayor. Describo un procedimiento en BASIC, apropiado para trabajarse con microcomputadoras, para calcular la distancia y la dirección inicial, dada la latitud y longitud de cada uno de los dos puntos.

When ecological data consist of pairs of geographic locations (e.g., locations of animals at two different times), it may be desirable to ask what the shortest distance is between the two points and what the initial compass direction is from the first point to the second. Several routes may be followed in traveling from one point to another on the earth's surface, but the shortest route is the length of an arc of a great circle. A great circle is the largest circle that can be drawn on the surface of a sphere; it is formed by the intersection of the surface of a sphere with a plane passing through the center of the sphere. Considering the earth to be a sphere (which may practically be done for typical navigational purposes), the equator is a great circle, as is any meridian line. (An interesting characteristic is that the compass direction of a great circle route changes along the route and may take the traveler farther north or farther south than either the starting or ending point of the travel.) Determining great-circle distance and direction requires solving for a side and an angle of a spherical triangle (e.g., see U.S. Naval Oceanographic Office 1975, pp. 583ff, or other references on navigation or trigonometry), and the fundamental mathematics have been previously described in the context of bird-banding data (Zar and Southern 1977). A listing of a BASIC program to effect the following calculations, is available from the author at no charge.

COMPUTING GREAT-CIRCLE DISTANCE

The shortest distance between points 1 and 2 on the earth's surface is

$$d = \arccos\{\cos(\mathrm{Dlat}) - [1 - \cos(\mathrm{Dlong})]\cos(\mathrm{lat}_1)\cos(\mathrm{lat}_2)\}$$

where

$\text{Dlat} = \text{lat}_1 - \text{lat}_2$
$\text{Dlong} = \text{long}_1 - \text{long}_2$
lat_i = latitude of point i
long_i = longitude of point i

and where south latitudes and east longitudes are considered to be negative numbers. Both the cosine and sine of an angle are functions built into the BASIC language: COS(x) is the cosine of x and SIN(x) is the sine of x. But trigonometric computation in BASIC employs radians, not degrees, so each input datum must be divided by 57.2958 (which is $180/\pi$, the number of degrees in a radian). The arccosine of y—also termed the inverse cosine of y and abbreviated as arccos(y) or $\cos^{-1}(y)$—is the angle that has y as its cosine. Normally, BASIC does not have an arccosine routine built in, but typically the arctangent function is available and we may employ the following relationship, using the BASIC functions for arctangent (ATN) and square root (SQR):

$$\arccos(y) = 1.57080 - 2*\text{ATN}(y/(1 + \text{SQR}(1 - y*y)))$$

(Lien 1986, p. 826).

The distance thus computed is in terms of the arc of a great circle, measured in radians. Multiplying y by 57.2958 will, therefore, give the distance in degrees. One degree of such an arc on the earth's surface is 60 international nautical miles, which is 69.0468 statute miles, or 111.12 kilometers.

COMPUTING GREAT-CIRCLE DIRECTION

The initial compass direction of a great-circle route may be obtained as follows. We first determine the angle

$$a = \arccos\{[\sin(\text{lat}_2) - \cos(d + \text{lat}_1 - 1.5708)]/\cos(\text{lat}_1)/\sin(d) + 1\}$$

where a is in radians (and, therefore, $57.2958a$ is the direction in degrees). The correct direction is either a or the mirror image of a along the north–south axis (i.e., either this direction, in degrees, or 360° minus this direction). We may ascertain which it is by examining Dlong: if either

$$|\text{Dlong}| < 180° \text{ (i.e., 3.14159 radians) and Dlong} < 0° \text{ (0 radians)}$$

or

$$|\text{Dlong}| > 180° \text{ and Dlong} > 0°$$

then set a equal to 360° (6.28319 radians) minus a.

Researchers who work with directional data should be aware that common statistical methods (e.g., for mean, standard deviation, and comparing samples—such as by t-testing or analysis of variance) are not applicable to data coming from a circular scale of measurement, such as compass directions (Zar 1984, pp. 422–469).

LITERATURE CITED

LIEN, D. A. 1986. The BASIC handbook, 3rd ed. CompuSoft Publishing, San Diego, California.

U.S. NAVAL OCEANOGRAPHIC OFFICE. 1975. American practical navigator. (Originally by Nathanial Bowditch), U.S. Government Printing Office, Washington.

ZAR, J. H. 1984. Biostatistical analysis, 2nd ed. Prentice-Hall, Inc., Englewood Cliffs, New Jersey.

———, AND W. E. SOUTHERN. 1977. Computer determination of flight distance and direction, with statistical analysis and mapping. Pp. 140–153, *in* Proceedings of the Colonial Waterbird Group Conference, Northern Illinois University, DeKalb, Illinois.

Received 14 Oct. 1988; accepted 22 May 1989.