Studies in Avian Biology No. 6:436–440, 1981.

IMPROVED POPULATION ESTIMATES THROUGH THE USE OF AUXILIARY INFORMATION

DOUGLAS H. JOHNSON[1]

ABSTRACT.—When estimating the size of a population of birds, the investigator may have, in addition to an estimator based on a statistical sample, information on one of several auxiliary variables, such as: (1) estimates of the population made on previous occasions, (2) measures of habitat variables associated with the size of the population, and (3) estimates of the population sizes of other species that correlate with the species of interest. Although many studies have described the relationships between each of these kinds of data and the population size to be estimated, very little work has been done to improve the estimator by incorporating such auxiliary information. A statistical methodology termed "empirical Bayes" seems to be appropriate to these situations. The potential that empirical Bayes methodology has for improved estimation of the population size of the Mallard (*Anas platyrhynchos*) is explored. In the example considered, three empirical Bayes estimators were found to reduce the error by one-fourth to one-half of that of the usual estimator.

The United States Fish and Wildlife Service (FWS) is charged by law with the authority and responsibility for migratory birds within the nation. Many species are protected by joint treaties with other nations: Great Britain (for Canada), Mexico, the Soviet Union, and Japan. One particular concern of the Fish and Wildlife Service is the regulation of hunting on game species. By late summer each year, regulations governing the hunting season during the subsequent fall and winter must be promulgated and published in the Federal Register.

In order to develop regulations that are consistent with the welfare of the game species, the FWS collects certain kinds of information about the status of those species (Martin et al. 1979). For waterfowl, which are of high interest to millions of hunters, the FWS each May conducts a survey of the population throughout the major breeding areas of North America. These surveys are done in cooperation with the Canadian Wildlife Service and various states and provinces. The survey is a complicated sample survey design (Martin et al. 1979), one sample unit being the transect, a linear route along which an aircraft is flown. Waterfowl are counted, according to species, within 0.2 km (⅛ mile) on either side of the aircraft. These counts are adjusted by the area covered, and by independently derived visibility rates, to estimate the density of waterfowl, by species, along each transect.

The sample counts are subject to fairly large variances, as well as possible biases. Although accurate population estimates are desired, improved precision through increased sample size is difficult to attain, because of the cost, time, and personnel requirements of the May surveys.

The purpose of this preliminary report is to examine the efficacy of a statistical methodology known as empirical Bayes for improving estimators of waterfowl density through the use of auxiliary information. The empirical Bayes methodology will be briefly surveyed. The kinds of auxiliary information considered are: (1) estimated population densities of the species of interest in previous years; (2) information on habitat variables that correlate with the density of the species; and (3) estimated densities of other species in the particular year.

METHODS

EMPIRICAL BAYES ESTIMATION

Assume we have a recurring problem of estimating a location parameter θ, for example, the average density of Mallards in eastern North Dakota. We have a statistic X, perhaps the average Mallard density of a sample of k transects, whose distribution depends on θ via the probability density function $f(x \mid \theta)$. Suppose that the situation recurs with various unknown values of θ. Let the distribution of θ be described by the probability density function $g(\theta)$. Suppose we have a sequence of n such situations, with observed statistics $x_1, x_2, \ldots, x_n$ and corresponding parameter values $\theta_1, \theta_2, \ldots, \theta_n$. We want to estimate the current value, θ_n; the current as well as previous statistics $x_1, \ldots, x_n$ are known to us.

The problem can be addressed from three points of view (Krutchkoff 1969). The *classical* approach incorporates the fact that X_n is sufficient for θ_n; therefore only the data for the current situation are used to estimate θ_n. For example,

$$\hat{\theta}_n = x_n.$$

This estimator is unbiased and has variance σ^2/k, where σ^2 is the variance of a single transect and k the number of transects that comprise the mean.

A strictly *Bayesian* approach would require that $g(\theta)$ be known *a priori*. The posterior distribution is then

$$p(\theta_n \mid x_n) = \frac{f(x_n \mid \theta)g(\theta)}{\int f(x_n \mid \theta)g(\theta)\, d\theta}.$$

A point estimator of θ_n can be taken as the mean of the posterior distribution:

$$\tilde{\theta}_n = E(\theta_n \mid x_n) = \frac{\int \theta f(x_n \mid \theta)g(\theta)\, d\theta}{\int f(x_n \mid \theta)g(\theta)\, d\theta}.$$

[1] U.S. Fish and Wildlife Service, Northern Prairie Wildlife Research Center, Jamestown, North Dakota 58401.

TABLE 1
DATA USED TO DEVELOP EMPIRICAL BAYES ESTIMATOR BASED ON PREVIOUS MALLARD DENSITIES

(1) Year (n)	(2) θ_n	(3) Sample 1	(4) Sample 2	(5) Sample 3	(6) x_n	(7) $s_n^2/3$	(8) $s^2/3$	(9) $\bar{x}$	(10) t^2	(11) $\hat{\theta}_n$
1958	11.0619	9.5162	15.4012	2.0451	8.9875	14.9353	14.9353	—	—	—
1959	3.9925	2.4834	5.2589	1.7530	3.1651	1.1405	8.0379	8.9875	—	—
1960	6.9193	8.3266	8.7649	8.8901	8.6605	0.0292	5.3683	6.0763	16.9502	8.0389
1961	6.4743	3.7140	8.1355	4.6994	5.5163	1.7960	4.4752	6.9377	10.7011	5.9354
1962	7.7613	7.7818	3.7898	10.0810	7.2175	3.3779	4.2558	6.5824	7.6392	6.9903
1963	12.5712	6.8975	13.2644	20.1620	13.4413	14.6701	5.9915	6.7094	5.8101	10.0236
1964	7.8600	5.2316	22.9385	2.4146	10.1949	41.2611	11.0300	7.8314	12.2012	9.0727
1965	8.9054	4.8844	10.4667	7.6523	7.6678	2.5969	9.9759	8.1690	10.9657	7.9066
1966	13.0822	7.3489	23.4924	7.9513	12.9309	27.9167	11.9693	8.1064	9.4306	10.2325
1967	9.0331	10.0919	10.3212	10.8092	10.4074	0.0447	10.7768	8.6424	10.8380	9.5274
1968	9.0470	6.4044	11.7682	13.7295	10.6340	4.7930	10.2329	8.8189	9.9453	9.7135
1969	8.2463	7.9337	8.0597	10.2578	8.7504	0.5694	9.4276	8.9839	9.2502	8.8683
1970	13.0868	7.5113	8.3892	20.8268	12.2424	18.4871	10.1245	8.9645	8.4139	10.4522
1971	13.0103	11.3584	9.2452	8.1886	9.5974	0.8683	9.4633	9.2166	8.5392	9.3972
1972	9.9792	25.1220	8.3740	7.2109	13.5690	33.4809	11.0645	9.2438	7.8927	11.0446
1973	8.8084	3.4951	6.6724	4.4483	4.8719	0.8861	10.4283	9.5322	8.5761	7.4292
1974	7.9099	4.6781	14.2320	3.6898	7.5333	11.2995	10.4796	9.2409	9.3617	8.4352
1975	7.9809	10.9896	12.3149	5.5662	9.6236	4.2619	10.1341	9.1404	8.9482	9.3670
1976	7.6175	9.3455	9.8446	11.4324	10.2075	0.3959	9.6216	9.1673	8.4348	9.6532
1977	4.0781	3.6468	6.2517	2.1881	4.0289	1.4126	9.2112	9.2220	8.0231	6.8045
1978	9.0618	7.7069	5.0626	6.9611	6.5769	0.6196	8.8020	8.9624	8.9493	7.7598
1979	12.0648	9.3060	8.7586	5.0049	7.6898	1.8272	8.4850	8.8488	8.7728	8.2596

The strictly Bayesian approach also ignores the earlier observations $x_1, \ldots, x_{n-1}$; instead it is necessary to assume that the prior distribution is completely specified.

In the *empirical Bayes* approach, we begin with the Bayes estimator $E(\theta_n | x_n)$, which is given in terms of the unknown prior distribution $g(\theta)$, and estimate it instead in terms of the data $x_1, x_2, \ldots, x_n$. There are several ways of doing this, each resting on different assumptions.

The superiority of empirical Bayes estimators was first suggested by Stein (1955) and James and Stein (1961), who considered the problem of estimating $n \geqslant 3$ independent normal means, each with variance one, and a quadratic loss function. For the set of n means, the maximum likelihood estimator X_n was inferior to $X_n[1 - b/(a + \Sigma X_i^2)]$, where b and a are selected constants and the summation is over all X's. Stein's procedure essentially shrinks the estimator away from the observed mean toward zero. Lindley (1962) recommended instead that they be shrunk toward the mean of all X's, and proposed the estimator

$$X_n[1 - (n - 3)/\Sigma(X_i - \bar{X})^2] + \bar{X}(n - 3)/\Sigma(X_i - \bar{X})^2.$$

Stein's estimator is a weighted average of the observed mean and zero; Lindley's estimator is a weighted average of the observed mean and the overall mean.

A wealth of estimators appropriate to more general situations have also been developed, and comparison with standard estimators has demonstrated their worth (e.g., Efron and Morris 1975). Despite the theoretical justification of empirical Bayes methods, their use has not been widespread.

THE EXAMPLE—MALLARDS IN EASTERN NORTH DAKOTA

Although numerous waterfowl species are counted during the May waterfowl surveys, the Mallard duck receives especial attention because of its abundance and prized status by hunters. For the immediate purpose of exploring alternative estimation procedures, this report treats only Mallard densities, and only in eastern North Dakota (FWS Strata 45 and 46 [Martin et al. 1979]).

Estimates are available annually 1958 to 1979. In each year a number (varying between 7 and 15) of transects were run in eastern North Dakota. To illustrate the empirical Bayes estimators developed here, it is desirable to know the "true" population parameter, against which the performance of various estimators can be judged. For our example, the average density of Mallards in all transects during a given year will be considered the true parameter. These values (θ) are given in the second column of Table 1. We randomly selected three of the transects to use as sample data; independent samples were drawn each year. The sample Mallard densities are given in columns 3–5 of Table 1.

The Mean Square Error (MSE) criterion will be used for comparing estimators. The MSE measures the average "closeness" of the estimator to the parameter being estimated. If we have n situations in which we develop two estimators E_i and F_i of an unknown parameter P_i ($i = 1, 2, \ldots, n$), then

$$\text{MSE}(E) = \sum_{i=1}^{n} (E_i - P_i)^2/n,$$

TABLE 2
DATA USED TO DEVELOP EMPIRICAL BAYES ESTIMATOR BASED ON WETLAND CONDITIONS

(1) Year (n)	(2) x_n	(3) $s_n^2/3$	(4) $s^2/3$	(5) W_n	(6) a	(7) b	(8) $\hat{\theta}_n(W)$	(9) Z_n^2	(10) $\hat{\hat{\theta}}_n(W)$	(11) θ_n
1958	8.9875	14.9353	14.9353	214.9	—	—	—	—	—	11.0619
1959	3.1651	1.1405	8.0379	88.5	—	—	—	—	—	3.9925
1960	8.6605	0.0292	5.3683	340.4	−.9115	.0461	14.7809	—	—	6.9193
1961	5.5163	1.7960	4.4752	64.5	2.2498	.0218	3.6559	12.7967	5.0343	6.4743
1962	7.2175	3.3779	4.2558	229.8	3.4757	.0175	7.4972	5.1959	7.3434	7.7613
1963	13.4413	14.6701	5.9915	357.1	3.4628	.0173	9.6406	4.7160	11.3146	12.5712
1964	10.1949	41.2611	11.0300	148.7	2.5472	.0245	6.1904	4.9667	7.4337	7.8600
1965	7.6678	2.5969	9.9759	303.3	3.7248	.0215	10.2458	7.3944	9.1484	8.9054
1966	12.9309	27.9167	11.9693	448.5	3.9539	.0190	12.4754	10.0027	12.6828	13.0822
1967	10.4074	0.0447	10.7768	480.5	3.8364	.0197	13.3022	7.5833	12.1066	9.0331
1968	10.6340	4.7930	10.2329	250.9	4.4417	.0164	8.5565	5.5348	9.2857	9.0470
1969	8.7504	0.5694	9.4276	495.8	4.6773	.0162	12.7093	6.7555	11.0567	8.2463
1970	12.2424	18.4871	10.1245	625.1	5.3669	.0126	13.2432	8.7660	12.7788	13.0868
1971	9.5974	0.8683	9.4633	452.6	5.5794	.0117	10.8748	5.7963	10.3896	13.0103
1972	13.5690	33.4809	11.0645	485.9	5.6382	.0112	11.0803	5.5015	11.9068	9.9792
1973	4.8719	0.8861	10.4283	221.3	5.4733	.0112	7.9519	5.3191	6.9115	8.8084
1974	7.5333	11.2995	10.4796	575.5	4.9832	.0131	12.5222	6.3527	10.6393	7.9099
1975	9.6236	4.2619	10.1341	539.0	5.5687	.0105	11.2282	7.1172	10.5662	7.9809
1976	10.2075	0.3959	9.6216	526.8	5.6950	.0099	10.9103	6.6681	10.6226	7.6175
1977	4.0289	1.4126	9.2112	220.7	5.7385	.0097	7.8793	6.1458	6.3384	4.0781
1978	6.5769	0.6196	8.8020	317.4	5.2116	.0106	8.5760	6.3954	7.7347	9.0618
1979	7.6898	1.8272	8.4850	487.9	5.0714	.0107	10.2919	6.4452	9.1686	12.0648

and E is a better estimator of P than F is if MSE (E) < MSE (F). Mathematically, the MSE equals the variance of an estimator plus the square of its bias.

RESULTS

Suppose in a given year n, the true density of Mallards in eastern North Dakota is θ_n, that value having resulted as a random outcome of a process with probability density function $g(\theta)$. We have an estimator of θ_n, given by X_n, which we assume is normally distributed with mean θ_n and variance σ^2/k. That is,

$$X_n \sim N(\theta_n, \sigma^2/k)$$

In the present example X_n is the estimated density of Mallard pairs based upon a sample of k transects in eastern North Dakota. X_n is unbiased and its variance σ^2/k is estimated by the sample variance S_n^2/k, where k is the sample size in that year. The mean of the three samples, x_n, is given in column 6 of Table 1. The sample variance of this mean is presented in column 7. The accuracy of the classical estimator can be evaluated by comparing columns 2 and 6. The Mean Square Error of the classical estimator for all 22 years of data is

$$\Sigma\ (x_n - \theta_n)^2/22\ .$$

Thus MSE (X_n) = 4.42 for all years. We will use as a test period the years 1968–79, permitting 10 years of baseline data to be used to develop the procedure. The MSE during the test period is 6.53.

AN ESTIMATOR BASED ON PREVIOUS COUNTS

A simple empirical Bayes estimator may be obtained by assuming that the process that generated θ_n was itself normal, with unknown mean θ and unknown variance τ^2:

$$\theta_n \sim N(\theta, \tau^2).$$

Then the empirical Bayes estimator is a weighted average of the current X_n and the mean of the previous X's, $\bar{X} = (X_1 + X_2 + \ldots + X_{n-1})/(n - 1)$. The weights are simply the reciprocals of the respective variances.

$$\hat{\theta}_n = \frac{X_n k/S^2 + \bar{X}/t^2}{k/S^2 + 1/t^2}. \tag{1}$$

where $t^2 = \hat{\tau}^2 = \Sigma(X_i - \bar{X})^2/(n - 2)$ and S^2 is the pooled within-year variance estimator.

This empirical Bayes estimator involves the current year's estimate, X_n, and the average of the Mallard densities from previous years, $\bar{X}$. These cumulative averages are given in column 9 of Table 1. The variance among the previous years' Mallard densities, which is used in the weighting of the cumulative averages, is shown in column 10. The simple empirical Bayes estimate, from Equation 1, is shown in column 11.

Comparing columns 2, 6 and 11, it is seen that the empirical Bayes estimator shrinks the sample mean, X_n, toward the cumulative mean, $\bar{X}$. This shrinkage on the average tends to produce an estimate closer to the true value, θ_n. The Mean Square Error for this estimator is 3.71 for all years and 4.48 for 1968–79 test period. The MSE for the test period is 31.4 percent lower than 6.53, the value for the classical estimator.

An Estimator Based on Wetland Conditions

A waterfowl biologist might balk at the procedure described above, despite the clear gain in accuracy it affords, because it includes averages of Mallard densities from all previous years. Biologists recognize that in some years the prairies are wet and the ponds are full, but in other years the prairies and the ponds are dry. Mallards are far more common in North Dakota during wet years than dry years; the correlation between Mallard density and pond index, also measured each May, for 1958–79 is 0.555. Accordingly, biologists would be reluctant to base an estimator of Mallard density during a wet year upon a cumulative mean involving dry years. The estimator proposed in this section overcomes this objection by incorporating information about wetland habitat conditions.

Suppose that the Mallard density in eastern North Dakota is related to the pond index W, in a particular year j, according to

$$\theta_j = \alpha + \beta W_j + \epsilon_j,$$

where $E(\epsilon_j) = 0$, $V(\epsilon_j) = \mu^2$. From the X's and W's of previous years, we can estimate α, β, and μ^2, by a, b, and m^2, respectively. The regression estimator of θ_n is thus given by

$$\hat{\theta}_n(W) = a + bW_n.$$

This estimator can be used in combination with the sample estimate in the current year according to:

$$\hat{\hat{\theta}}_n = \frac{X_n k/S^2 + (a + bW_n)/Z^2}{k/S^2 + 1/Z^2}.$$

In this formula, Z^2 is the variance of an individual value of θ predicted from W:

$$Z^2 = m^2[1 + (n-1)^{-1} + (W_n - \bar{W})^2/\Sigma(W_i - \bar{W})^2] \quad (2)$$

where m^2 is the residual variance and is equal to

$$\begin{aligned} m^2 &= \Sigma[X_i - \hat{\theta}_i(W)]^2/(n-3) \\ &= [\text{Var}\, X_i - b^2 \text{Var}\, \hat{\theta}_i(W)](n-2)/(n-3) \\ &= \text{Var}\, X_i(1 - r^2)(n-2)/(n-3) \\ &= t^2(1 - r^2)(n-2)/(n-3). \end{aligned}$$

Note that r^2 is the squared simple correlation coefficient between pond index (W) and Mallard density (X).

Returning to the 1958–79 data for Mallards in eastern North Dakota, we now consider the improvement possible by including information about wetland conditions. Table 2 displays the pertinent information. Columns 2 and 3 contain the sample mean and its variance for a particular year. The pond index is given in column 5. Columns 6 and 7 provide the intercept and slope for estimating Mallard density from pond index, based on the data from years prior to the current one. The estimate of θ, based on a, b, and W_n, is given in column 8, with associated variance in column 9. The empirical Bayes estimator is shown in column 10, to be compared to the true value in column 11.

The Mean Square Error of this estimator is 3.91 for all years and 5.05 for the 1968–79 test period. This estimator thus offers a 23% improvement in MSE over the classical one, but does not perform quite as well as the empirical Bayes estimator based on the overall mean of mallard densities.

An Estimator Based on Other Species

In addition to the Mallard, five other dabbling ducks are common in the prairies of eastern North Dakota. These are Gadwall (*Anas strepera*), American Wigeon (*A. americana*), Blue-winged Teal (*A. discors*), Northern Shoveler (*A. clypeata*) and Pintail (*A. acuta*). These six species tend to fluctuate together; the multiple correlation coefficient between Mallard density and the densities of other species is $R^2 = 0.62$. This value is appreciably higher than the R^2 between pond index and Mallard density, $R^2 = 0.31$.

The reasoning above suggests that the sample densities of other species in a particular year might be used to develop an estimator of the Mallard density that year. This estimator could be combined in an empirical Bayes manner with the sample Mallard density. The following result, incorporating only one other species, indicates the potential power of the method.

The single species most closely correlated with Mallard densities in Strata 45 and 46 during 1958–79 was the Pintail, with $r = 0.61$. A regression equation relating Mallard density (θ_n) to Pintail density from all transects (P_n) is given by

$$\hat{\theta}_n(P) = 5.7922 + 0.3421 P_n.$$

Unlike previous analyses, this predictive equation was developed from the entire 22-year data set, rather than sequentially year by year. Table 3 displays the Mallard densities estimated from Pintail densities (column 5), the weighting factors obtained analogously to equation 2 (column 4), and the resulting empirical Bayes estimator (column 6).

TABLE 3
DATA USED TO DEVELOP EMPIRICAL BAYES ESTIMATOR BASED ON PINTAIL DENSITIES

(1) Year	(2) P_n	(3) θ_n	(4) Z_n^2	(5) $\hat{\theta}_n(P)$	(6) $\hat{\hat{\theta}}_n(P)$
1958	8.3321	11.0619	6.1140	8.6426	8.7428
1959	1.1431	3.9925	7.0121	6.1833	4.7771
1960	12.4357	6.9193	6.2156	10.0465	9.3028
1961	6.3318	6.4743	6.2263	7.9583	6.5375
1962	12.3693	7.7613	6.2104	10.0237	8.3586
1963	7.4014	12.5712	6.1531	8.3242	10.9168
1964	9.8798	7.8600	6.0999	9.1721	9.5363
1965	12.7789	8.9054	6.2443	10.1639	9.2030
1966	19.8332	13.0822	7.5264	12.5771	12.7137
1967	10.2286	9.0331	6.1055	9.2914	9.6950
1968	3.8462	9.0470	6.5137	7.1080	8.4795
1969	14.8464	8.2463	6.4834	10.8712	10.0070
1970	14.6725	13.0868	6.4590	10.8117	11.3689
1971	11.9692	13.0103	6.1816	9.8869	9.7725
1972	12.3579	9.9792	6.2095	10.0198	11.2956
1973	4.2776	8.8084	6.4521	7.2556	6.3445
1974	8.8351	7.9099	6.1025	8.8147	8.3431
1975	8.3193	7.9809	6.1144	8.6382	9.0090
1976	4.0097	7.6175	6.4897	7.1639	8.3899
1977	1.1215	4.0781	7.0168	6.1759	5.2476
1978	12.0574	9.0618	6.1876	9.9170	8.5382
1979	10.8530	12.0648	6.1235	9.5050	8.7438

The Mean Square Error of this estimator is 2.61 for all years and 3.26 for the test period. This latter value represents a 50% decrease in MSE compared to that of the ordinary mean. Although the estimator based on Pintail densities is not directly comparable to the others, because data from all years were used to develop each year's predictor, the potential worth of the estimator is nonetheless evident. Other species in addition to the Pintail could be used in an empirical Bayes manner, but I suspect a direct multivariate approach might prove more productive.

In a multivariate empirical Bayes approach the six individual species could be considered together as a 6-variate vector. Interest lies in estimating the entire vector, and the methods outlined in Efron and Morris (1972) can be used to develop empirical Bayes estimators that are better than the classical ones. Efron and Morris (1972:341) suggested that the multivariate approach will be preferable to a component-by-component univariate procedure if the variables are relatively highly correlated. This condition seems to be readily satisfied with the waterfowl density values.

DISCUSSION

This report has addressed the problem of improving the accuracy of waterfowl population estimates without additional sampling effort and the associated costs. The technique has been to invoke auxiliary information to develop a prior estimate of Mallard density. This prior value is combined with the estimate obtained by sampling to form an empirical Bayes estimate.

For the example considered here, an ordinary EB estimator, which uses the mean of earlier years as a prior estimate, was found to reduce the MSE by 31 percent for the 1968–79 test period. The implication is that the accuracy of the estimator of Mallard density in eastern North Dakota could be substantially improved simply through the use of EB estimation. Alternatively, the current precision could be maintained, but costs reduced, by sampling fewer transects and employing EB procedures.

We also considered an EB estimator based upon the relationship of Mallard density to an index of wetland conditions. This estimator proved, in the example, to be better (23% in MSE) than the classical one, but, perhaps surprisingly, it was not quite as accurate as the previous EB estimator.

The third estimator examined was based on the density of Pintails in each year. The predictive equation was derived from the entire 22-year sample, unlike the other estimators which used formulas incorporating only data from prior years. Thus the 50% reduction in MSE is not exactly comparable to the improvements obtained by the other estimators, but it illustrates the potential of the method.

The theory of empirical Bayes methods has existed for a quarter of a century. Despite a fairly well developed theory, relatively few practical applications have been made thus far, but this situation seems to be changing. I anticipate that EB procedures will have widespread uses in many fields before long.

Empirical Bayes procedures seem particularly promising for surveys of bird populations. Many surveys are conducted regularly, usually annually, accuracy is highly desired, and the sample data are often expensive or difficult to obtain. More research must be done to determine those problems the procedures can most profitably address. I suggest that EB estimators will be of greatest value in regular surveys of less common species, those that are the most difficult to measure, or those whose density can be best predicted from other available information.

ACKNOWLEDGMENTS

I appreciate the efforts of Richard S. Pospahala and Edgar L. Ferguson, who provided the data used to exemplify the methods, and of Stella G. Machado and Fred L. Ramsey, who commented on earlier drafts of the paper.